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THEORY OF THE COULOMB SCATTERING OF FAST ELECTRONS IN THE EARTH'S OUTER RADIATION ZONE

by B. M. Tverskoy

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THEORY OF THE COULOMB SCATTERING OF FAST ELECTRONS IN THE EARTH'S OUTER RADIATION ZONE

(K teorii Kulonovskogo rasseyaniya bystrykh elektronov vo vneshney zone radiatsii Zemli)

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by B. M. Tverskoy

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ABSTRACT

Coulomb scattering of fast electrons in a cold, fully-ionized plasma is being investigated in symmetric magnetic traps. Derived is the kinetic equation for the distribution function in the symmetry plane. A general solution is given for the corresponding problem with initial conditions. The solution is brought to the numerical results for the case of a linear and approximately a point dipole at cold plasma's constant density. It is shown, that the death of electrons is then basically due to energy losses at collisions, while practically no part is played by scattering in the plugs. Lifetime of various angular harmonics is computed, It resulted, that the first harmonic maintains the initial angular distribution during the time, comparable with the electron lifetime. The remaining harmonics damp quickly as a result of collisions, while the corresponding particles acquire an isotropic distribution by angles with practically no loss of energy. On the basis of the results obtained, the relationship of the settled distribution with the angular distribution of the sources is being discussed.

* * *

COVER-TO-COVER TRANSLATION

The explanation of loss of electrons by escape constitutes one of the basic questions of the theory of the Earth's outer radiation belt. One may expect that the basic mechanism of death is the Coulomb scattering in the cold plasma. As to the so-called "magnetic scattering" [1] (sharp variations of the magnetic moment at electron interaction with shock wave-type perturbations), a concrete examination of similar processes [2 – 3] shows that there basically takes place an increase in energy and ~~~ in the magnetic moment of particles, and consequently an extension of their lifetime.

The zone's stability also indicates that the death of electrons is much rather connected with the slow processes of the Coulomb scattering type. Therefore it is rightful to examine the problem in a set up, when the Coulomb scattering is the fundamental factor limiting the lifetime.

A semi-qualitative examination of such problem was effected by Cristofilos [4]. He assumed, that the death of electrons is basically linked with scattering in the plugs. The sharp anisotropy of the angular distribution at low altitudes leads to a systematic increase of particle's longitudinal velocity at collisions, and to a deepening in the atmosphere of the reflection point.

It is possible that such process is essential for particles, penetrating through altitudes where the density varies sharply with the distance. But for most of particles, oscillating at great heights, where the density is nearly constant, the increase in longitudinal velocity in the plugs is being compensated by the diffusion effect.

The most reliable study of the Coulomb scattering is linked with the solution of the kinetic equation. In case of the outer zone such problem is significantly simplified because of the following :

a) the concentration of fast electrons is small in comparison with that of the cold plasma, with the result that the equation is linear ;

b) thanks to Sun's ultraviolet radiation, the cold plasma is fully ionized, so that ionization losses, difficult to account for in a kinetic equation, may be neglected ;

c) the conditions of drift approximation are well fulfilled ;

d) the particle's lifetime in the trap τ_0 is much greater than the oscillation time between plugs τ_1, so that averaging in times of the order of τ_1 may be effected ;

e) the fundamental mass of electrons has non-relativistic velocities, while the Debye radius is small in comparison with the Larmor radius, so that the scattering may be described by the Rutherford formula ;

f) The electric and magnetic fields, appearing on account of fast particle drift, may be neglected ;

g) the trap is axially-symmetric and has a symmetry plane.

The subsequent computations are based upon these assumptions. First, we shall obtain a general equation, valid for any traps satisfying the conditions a) through g), and we shall examine the general relationship of problem's solution with the initial conditions. Then, the result obtained will be concretized for the case of a linear and point dipole.

I. KINETIC EQUATION

The Coulomb diffusion across the field is immaterial in the examined approximation, since during collision time, the diffusion shifting will be only of the order of Larmor radius.* In the current work, phenomena taking place in the field's tube of force will be examined. Its cross section in the symmetry plane may be postulated equal to the unity, without prejudice to the generality.

* Parker [5] proposed recently an interesting diffusion mechanism across the field at the expense of third invariant's non-preservation during magnetic storms. However, his computations arouse doubts and in the following, after making it more precise, the Parker diffusion will be proposed for inclusion in the examined scheme.

The distribution function of fast electrons f satisfies the equation

$$\frac{\partial f}{\partial t}+V_{\parallel}\frac{\partial f}{\partial l}-M\frac{dH}{\partial l}\frac{\partial f}{\partial V_{\parallel}}=\frac{1}{\tau_s(V,\,l)}\left\{V\frac{\partial f}{\partial V}+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial f}{\partial\theta}\right\}+q, \quad (1)$$

where l is the coordinate along the line of force, calculated from the symmetry plane; $V_{\parallel}$ is the projection of velocity on the field; θ is the angle between the velocity and the line of force; $M = mV^2/2H$ is the magnetic moment; H is the magnetic field intensity;

$$\tau_s(V,\,l)=\frac{m^2V^3}{4\pi N(l)\,e^4\ln\frac{\pi}{\alpha_0}} \quad (2)$$

is the collision time, m and e are respectively the mass and the charge of the electron; $N(l)$ is the density of the number of particles of cold gas (which we assume to be once-ionized); α_0 is the scattering angle with an aiming distance equal to the Debye radius.

We designate by q the "sources' density", which includes all possible non-Coulomb and non-drift variations of the distribution function. The stability of the zone is the basis for separating q.

The left-hand term (1) is written in the drift approximation [6], while the collision operator $\hat{s}f$ is written in the Landau differential form [7], taking into account the preeminent character of scattering at small angles. The cold particles' thermal velocities are not taken into account.

The direct solution of (1) is difficult, since the equation describes two sharply distinct processes of equilibrium fixing — collision and particle mixing in the trap because of longitudinal velocity spread.

At arbitrary introduction in the trap of a group of particles, the distribution will not be stationary, generally speaking. Thus, if at the initial moment a particle cloud appeared near one of the plugs, it will begin to oscillate between plugs, gradually

expanding on account of longitudinal velocities' spread. In the end, the expansion will asymptotically lead to the settling of a stationary state (a similar problem for drift velocities is examined in [1]). If the spread is of the order of the period of particle oscillations between plugs, the mixing takes place during the time $\sim\tau \ll \tau_s$.

During the time $\Delta t \gg \tau_s$ the distribution function f satisfies with a high accuracy the collisionless drift equation. This means in the case of stationary processes, that f depends only on V and M, while the dependence on l is only manifest through M (or the dependence on H, which is the same. Here f is fully determined by the fixing of distribution in the symmetry plane.

It is easy to see that the colliding term clearly comprises the dependence on H. Therefore particles born during the time dt on account of diffusion may effect during the time $\sim\tau_1$ the oscillations corresponding to the mixing by the trap.

Therefore, fast oscillations with a small amplitude $\underline{\delta f}$ will be superimposed to low $\underline{f}$ variations.

Because of their smallness, δf amplitudes are not counted in the total balance of particles, but its influence upon $\partial f/\partial t$ may be excluded by a corresponding averaging, The final aim in averaging consists in the obtention of an equation for distribution functions in the symmetry plane.

The averaging is based upon the following principle : if the distribution of particles in the whole tube is known along V and M, the stationary distribution function $f_o(V, M)$ in the symmetry plane is uniquely determined by same, that stationary distribution being settled as a result of velocity dispersion during the time $\sim\tau_1$.

Let us introduce a new variable in place of M

$$\mu = \sin\theta \sqrt{\frac{H_o}{H}} \equiv \frac{\sin\theta}{\sqrt{h}}, \qquad (3)$$

which is an adiabatic invariant, just as M is. If $f_o = f_o(V, \mu)$, $f(V, \theta, H)$ is obtained by substituting in f_o the magnitude μ by

$\sin\vartheta/\sqrt{h}$ (with a precision to the oscillating addition $\sim \tau/\tau_5$

Let us determine the function $\psi(V, \mu)$ as the total number of particles with given V and μ in the whole tube. It is convenient to characterize the density of the sources by the function $Q(V, \mu)$, giving the number of particles with the assigned V and generated in 1 second in the total volume too.

The equilibrium condition for $\psi(V, \mu)$ will take the form

$$\frac{\partial\psi}{\partial t} = \left(\frac{\partial\psi}{\partial t}\right)_s + Q, \qquad (4)$$

where $(\partial\psi/\partial t)_s$ is the variation of the number of given type of particles in the tube, on account of collisions.

Function f and the colliding term $\hat{S}f$ are normalized to the element of solid angle in the velocity space

Such normalization is inconvenient, inasmuch as the spatial dependence is clearly outlined. However, this difficulty is easy to remove by transiting to normalization in the interval $d\mu$. Designating by the respective lower index such a normalization, we have for the arbitrary function φ:

$$\varphi_\mu(V, \mu, H, t)\, d\mu = \varphi_0(V, \theta, H, t) \cdot 2\pi \sin\theta d\theta.$$

Expressing θ through μ, we obtain the following transition formula from (3):

$$\varphi_\mu = \varphi_0(V, \theta(\mu), H, t)\frac{2\pi\mu h}{\sqrt{1-\mu^2 h}}. \qquad (5)$$

Taking into account that $f_0(V, \theta, H, t) = f_0(\mu, V, T)$, we obtain:

$$f_\mu(\mu, V, t) = \frac{2\pi\mu h}{\sqrt{1-\mu^2 h}} f_0, \qquad (6)$$

$$(\hat{S}f)_\mu = \frac{2\pi}{\tau_s}\left\{\sqrt{1-\mu^2 h}\frac{\partial}{\partial\mu}\mu\frac{\partial f_0}{\partial\mu} + \frac{h\mu}{\sqrt{1-\mu^2 h}}\left(V\frac{\partial f_0}{\partial V} - \mu\frac{\partial f_0}{\partial\mu}\right)\right\}. \qquad (7)$$

Integrating (7) along the tube, we obtain:

$$\frac{\partial \psi}{\partial t} = \frac{4\pi\mu}{\tau_0(V)} \left\{ \frac{P(\mu^2)}{\mu} \frac{\partial}{\partial \mu} \mu \frac{\partial f_0}{\partial \mu} + K(\mu^2) \left(V \frac{\partial f_0}{\partial V} - \mu \frac{\partial f_0}{\partial \mu} \right) \right\}, \tag{8}$$

where $\tau_0(V)$ is the collision time in the symmetry plane ($N = N_0$), but

$$P(\mu^2) = \int_0^{l(\mu^2)} \frac{N(l)}{N_0} \frac{dl}{h} \sqrt{1 - \mu^2 h}, \tag{9}$$

$$K(\mu^2) = \int_0^{l(\mu^2)} \frac{N(l)}{N_0} \frac{dl}{\sqrt{1 - \mu^2 h}}, \tag{10}$$

where $l(\mu^2)$ are the coordinates of the reflection point.

For the computation of $\psi_\mu(V, \mu)$, we shall take advantage of the expression for the given kind-particle flux through 1 cm^2 tube cross section in the symmetry plane. It is obvious that it is equal to (normalizing on $d\Omega$):

$$f_0(V, \mu, t) V \sqrt{1 - \mu^2}. \tag{11}$$

All particles with the given V and μ, will cross the symmetry plane once during the oscillations' $\tau_1(V, \mu)$ half-period.

Taking into account that

$$\tau_1(V, \mu) = \frac{2}{V} \int_0^{l(\mu^2)} \frac{dl}{\sqrt{1 - \mu^2 h}}, \tag{12}$$

and passing to normalization on $d\mu$, we shall have:

$$\psi_\mu(V, \mu, t) = 4\pi\mu f_0(V, \mu, t) R(\mu^2), \tag{13}$$

where

$$R(\mu^2) = \frac{V\tau_1}{2}$$

In case of constant density $N = N_0$

$$P_0(\mu^2) = \int_0^{l(\mu^2)} \frac{dl}{h} \sqrt{1 - \mu^2 h}, \qquad K_0(\mu^2) = R(\mu^2) = -2 \frac{dP_0(\mu^2)}{d\mu^2}. \tag{9a}$$

The equation for $f_0(V, \mu)$ is obtained from (4) after substituting (8) and (13). In case of constant density it has the form:

$$\frac{\partial f_0}{\partial t} = \frac{1}{\tau_0(V)}\left\{\sigma(\mu^2)\frac{1-\mu^2}{2\mu}\frac{\partial}{\partial\mu}\mu\frac{\partial f_0}{\partial\mu} - \mu\frac{\partial f_0}{\partial\mu} + V\frac{\partial f_0}{\partial V}\right\} + \frac{Q_\mu(V,\mu)}{4\pi\mu R(\mu^2)}, \quad (14)$$

where

$$\sigma(\mu^2) = \frac{2}{1-\mu^2}\frac{P_0(\mu^2)}{K_0(\mu^2)}. \quad (15)$$

As computations show, function $\sigma(\mu^2)$ for the most diversified traps has quite a similar form. Over the $0 \leq \mu^2 \leq 1$ - intercept, it is limited.

For a unique determination of $f_0(V, \mu)$, it is necessary to assign a boundary condition along μ. Particle death takes place for small μ in any trap, either on account of outflow from the trap, or because of the presence of absorbing surfaces. Designating by h_0 the value H/H_0, corresponding to the place of death, we obtain:

$$f_0\big|_{\mu^2 \leq \frac{1}{h_0}} = 0. \quad (16)$$

For given σ, Q and R, (14) may be fully investigated by standard methods of mathematical physics.

2. PROBLEM WITH INITIAL CONDITIONS : GENERAL THEORY.

For the determination of particle lifetime in the trap, and time of existence of various angular distribution harmonics, the problem with initial conditions without sources must be solved for the equation (14).

Taking advantage of the separation of variables method, let us seek the solution in the form

$$f_0 = e^{-pt}U(u)\,\Phi(\mu). \quad (17)$$

We obtain:

$$u\frac{dU}{du} = (\lambda - p\tau_0 u)\,U, \quad (18)$$

$$\frac{\sigma(\mu^2)(1-\mu^2)}{2\mu}\frac{d}{d\mu}\mu\frac{d\Phi}{d\mu} - \mu\frac{d\Phi}{d\mu} = -\lambda\Phi, \quad (19)$$

where λ is the separated constant; $u = V/V_0$; V_0 is the constant

introduced through dimension considerations.

From (18), it follows that

$$U(u) = u^{\lambda} e^{-\frac{p\tau_0(V)}{3}u^3}. \qquad (20)$$

By substituting the variable $\mu^2 = x$, equation (1) takes the form

$$\sigma(x)(1-x)\frac{d}{dx}x\frac{d\Phi}{dx} - x\frac{d\Phi}{dx} + \frac{\lambda}{2}\Phi = 0 \qquad (21)$$

and may be represented in a self-adjoint form.

On the basis of the general theory [8], solution of (21) must be searched for, satisfying the condition (16) and and the finiteness condition at $x = 1$. Hence determined is the discrete spectrum λ_n (while all $\lambda_n \geqslant 0$), and the corresponding assembly of eigenfunctions forms a complete system of a class of functions, bounded on the $[1/h_0, 1]$ intercept, and transforming to zero for $x = 1/h_0$

For the solution of the equation with the initial condition

$$f_0(u, \mu, 0) = \varphi(u, \mu)$$

the function $\varphi(u, \mu)$ must be expanded in series along $\Phi_n(\mu)$:

$$\varphi(u, \mu) = \sum_{n=0}^{\infty} \varphi_{n0}(u^3)\, \Phi_n(\mu^2). \qquad (22)$$

Let us represent $\varphi_{n_0}(u)$ with the aid of (20) in the form of a Laplace transformation along $p\tau_0/3$:

$$\varphi_{n0}(u^3) = (u^3)^{\frac{\lambda_n}{3}} \int_0^{\infty} \tilde{\varphi}_{n0}\left(\frac{p\tau_0}{3}\right) e^{-u^3\frac{p\tau_0}{3}}\, d\left(\frac{p\tau_0}{3}\right). \qquad (23)$$

Then

$$\varphi_n(u^3, t) = (u^3)^{\frac{\lambda n}{3}} \int_0^{\infty} \tilde{\varphi}_{n0}\left(\frac{p\tau_0}{3}\right) e^{-\frac{p\tau_0}{3}\left(u^3 + \frac{3t}{\tau_0}\right)}\, d\left(\frac{p\tau_0}{3}\right)$$

We find with the help of the well known formula of Laplace transformation theory :

$$\varphi_n(u^3, t) = \left(1 + \frac{3t}{\tau_0 u^3}\right)^{-\frac{\lambda_n}{3}} \varphi_{n0}\left(u^3 + \frac{3t}{\tau_0}\right). \qquad (24)$$

The second multiplier in (24) describes the particles' deceleration as a consequence of energy losses at collision, while the first one gives the decrease of the number of particles with the given distribution by angles because of scattering and death in plugs.

The retardation time from $V = V_0$ to zero is

$$t_l = \frac{\tau_0(V_0)}{3} = \frac{m^2 V_0^3}{12\pi N_0 e^4 \ln\frac{\pi}{\alpha_0}}. \qquad (25)$$

The characteristic existence time of the n-th angular harmonic relatively to collisions may be determined as the time t_n, during which the amplitude (24) diminishes twice. Assuming $u = 1$, we find:

$$t_n = t_e\left(2^{\frac{3}{\lambda_n}} - 1\right). \qquad (26)$$

In case of harmonics of high order $(\lambda_n \gg 3)$ $t_n \approx t_e \cdot 3/\lambda_n \ll t_e$, damping also is linked with scattering. The scattering doubly influences λ_n: firstly, it leads to death in plugs, secondly – it transfers the non-isotropic distribution by angles to the isotropic.

Thus, the problem amounts in fact to the finding of eigen-functions and eigen-values from (14) and (21). Because λ_n is being determined from the condition of finiteness at $x = 1$, the respective computations may be effected quite simply with the aid of a computer.

The approximate investigation of the problem may also be conducted analytically.

3. PROBLEM WITH INITIAL CONDITIONS: LINEAR AND POINT DIPOLE

In case of a linear dipole $\sigma(\mu^2) = 1$. In case of the

point dipole, it is given with a good precision (to 5 percent) by the formula:

$$\sigma(\mu^2) = 2\frac{2+\mu^2}{5+\mu^2}. \qquad (27)$$

At $\mu = 0$, the value $\sigma = 4/5$, and at $\mu = 1$, it is $\sigma = 1$. Thus one may postulate, that for a point dipole $\sigma = 1$ (with errors not exceeding 10 percent).

At the same time, equation (21) is hypergeometrical:

$$x(x-1)\Phi'' + (-1+2x)\Phi' - \frac{\lambda}{2}\Phi = 0 \qquad (28)$$

with parameters

$$\alpha = \frac{1+\sqrt{1+2\lambda}}{2}, \quad \beta = \frac{1-\sqrt{1+2\lambda}}{2}, \quad \gamma = 1. \qquad (29)$$

In case of the outer radiation zone, we may take for the absorbing surface the atmosphere boundary with a steep exponent (≈ 2000 km above the Earth's surface). To that boundary corresponds $h_0 \approx 30 \gg 1$. In this case the problem is easily solved by the method of consecutive approximations. In the zero approximation, we shall replace the condition (16) by the requirement of finiteness of $\Phi(x)$ at zero.

The solution of (28), finite at $x = 1$, has the form (9):

$$\Phi = F(\alpha, \beta, \gamma, 1-x), \qquad (30)$$

where F is a hypergeometrical function.

Since $\gamma - \alpha - \beta = 0$, Φ is finite at $x = 0$ only in case of degeneration in the polynome.

At the same time

$$\beta = -n \ (n = 0, 1, 2, \ldots). \qquad (31)$$

We shall postulate in the following approximation $\beta = -(n+\varepsilon)$ where $\varepsilon \ll 1$. Expanding the hypergeometrical series in powers ε, and limiting ourselves to zero and first terms, we shall have:

$$\Phi = F(n+1, -n, 1, 1-x) + (-1)^{n+1}\varepsilon \sum_{k=1}^{\infty} \frac{(2n+k)!\,(k-1)!}{[(n+k)!]^2}(1-x)^{n+k}.$$

Separating the diverging part of (32), we obtain

$$\Phi = F(n+1, -n, 1, 1-x) + (-1)^n \varepsilon (\ln x - C_n). \qquad (33)$$

It may be shown that $F(n+1, -n, 1, 1) = (-1)^n$. Substituting (33) in the boundary condition, we find:

$$\varepsilon_n = \frac{1}{\ln h_0 + C_n} (C_0 = 0, C_1 = 1, C_2 \approx 4, C_3 \approx 15, \text{ etc...}) \qquad (34)$$

Consequently, in the first approximation

$$\beta = -\left(n + \frac{1}{\ln h_0 + C_n}\right).$$

Hence

$$\lambda_n = 2n(n+1) + 2(2n+1)\frac{1}{\ln h_0 + C_n}. \qquad (35)$$

The zero harmonic has the form

$$\Phi_0 = 1 + \frac{\ln x}{\ln h_0}. \qquad (36)$$

The subsequent terms in the expansion by $1/h_0$ have the order $1/h_0$, $1/h_0^2$, etc..

The correction of the first approximation has a simple physical meaning, and it describes the particles' death in plugs. The comparative smallness of that effect is the result of the small "effective cross section" of the absorbing surface.

It is easy to compute with the aid of (35) the existence time of the various harmonics. In case of dipole, the basic damping mechanism of the upper harmonics is the scattering with the establishment of isotropy.

The table next page provides a compilation of five first conventional harmonics, and the corresponding values λ_n and t_n/t_e (at $n \geqslant 2$ the correction is immaterial).

Beginning with the second, all harmonics pass as a result of scattering to isotropic in a time, much lesser that the retardation time $t_e = \tau_0/3$.

As to the first harmonic, it keeps the initial distribution by angles during a time of the order of t_e. Nearly 10 percent of particles perish in the plugs (at $h_o = 30$).

n	$\Phi_n(x)$	λ_n	t_n/t_e
0	$1 + \ln x/\ln x_0$	$2/\ln h_0$	$2^{3/2} \ln h_0 - 1$
1	$1 - 2x + \ln x/1 + (\ln h_0)$	$4+6/(1+\ln h_0)$	0,5 at $h_0=30$
2	$1 - 6x + 6x^2$	12	0,2
3	$1 - 18x + 30x^2 - 20x^3$	24	0,13
4	$1-20x+90x^2-140x^3+70x^4$	40	0,07

Thus, it results that the basic mechanism of electron death in the outer radiation zone is in the energy losses at collisions (not taking into account the hypothetic "magnetic scattering").

4. ADDENDA TO THE THEORY OF THE ZONE'S STATIONARY STATE

As already noted above, the steadiness of the outer zone of radiation attests to the fact, that slow processes play the fundamental part in its formation. The accelerating mechanism supplies (feeds) the electrons in small portions. That is why the number of electrons "memorizing" the initial angular distribution communicated to them at acceleration, is relatively small.

As to the basic mass of electrons, forming the zone, it had time to undergo the forming effect of the Coulomb scattering, so that the angular harmonics with $n \geqslant 2$, passed into an isotropic distribution. One may expect with a great degree of probability, that the zero and the first harmonics predominate in the angular distribution of particles at the equator. In connection with that, the electron distribution function may be constructed in the zone according to data of the experiment.

Let the intensity, created by electrons with energies from E to $E + \Delta E$, be measured at two points, near the summit of a certain line of force. We search for the angular distribution

in the equator plane in the form of the sum of the two first harmonics, while in the case of a notable high course, the logarithmic correction

$$f_0(\mu^2) = a_1 + a_2(2\mu^2 - 1). \qquad (37)$$

may be neglected.

Passing from the angle θ_0 to θ ($\sin^2\theta_0 = \sin^2\theta/h$) and integrating along the angles, we obtain the density at the point h:

$$\nu(h) = 2(a_1 - a_2) + \frac{8}{3} a_2 \frac{1}{h}. \qquad (38)$$

Hence, and also from the experiment the relation $\varkappa = a_1/a_2$ is determined. The amgnitude a_1 is always positive. The sign of the magnitude $\varkappa$ determines the altitude course of intensity: at $\varkappa < 0$, the intensity increases with height, but at $\varkappa < 0$ — it drops.

Let us note that the death of particles in the plugs in the case of cold plasma's constant density, (N_0), cannot lead to a significant altitude course at great heights. The isotropic harmonic with small cut-out angles, has the form

$$\Phi_0 = 1 + \frac{\ln \mu^2}{\ln h_0}. \qquad (39)$$

Passing to densities, we obtain, that at great altitudes (at h = 30 ÷ 50) the variation of h from 1 (equator) to 2 decreases the density only by 10 to 20 percent).

If the true altitude course fits within these bounds, it may be the consequence of scattering in the plugs. But if the density variation of fast electrons along the line of force is more substantial (which apparently is the case in reality), the cause must be sought in the anisotropy of the angular distribution of the accelerating particles.

At the same time, to a positive altitude course corresponds a principal acceleration of the transverse motion, and to the negative — a longitudinal heating.

* * *

C O N C L U S I O N S

On the basis of the conducted investigation, the following conclusions may be drawn :

a) Amongst all possible manifestations of the Coulomb scattering the basic influence on the fast electrons' lifetime in the outer radiation zone is exerted by energy losses at collisions.

b) Scattering at low altitudes, and the death in plugs connected with it, result less significant (the death in plugs constitutes only 0.1 of the death on account of retardation, for $h_o = 30$.

c) The basic part of the zone's electrons is distributed between the isotropic and the first harmonic of the angular distribution.

d) The upper angular harmonics pass to isotropic in a lapse of time, much smaller than the electron's lifetime in the zone.

e) The electron lifetime t_e is entirely determined by its initial energy E, and it is

$$t_e = \frac{10^8}{N_o} E^{3/2} \text{sec} = \frac{3E^{3/2}}{N_o} \text{ years},$$

where E is measured in keV, and N_o — in $1/cm^3$.

The validity of these conclusions is limited by the assumptions about the absence of electron death on account of magnetic scattering, and on the constancy of the cold plasma. Although these assumptions are not arbitrary, and a series of arguments may be brought out to their advantage, it would be interesting to conduct an investigation free from the indicated limitations for the study of the dynamics of the outer zone.

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E N D

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MOSCOW STATE UNIVERSITY
in the name of
M. V. Lomonosov
Institute of Nuclear Physics.

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